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Hamiltonian theory used to analyze the properties of physical fields in the radiation regime

A PhD dissertation performed in the Center for Theoretical Physics PAS
and submitted to the Faculty of Physics at the University of Warsaw

Abstract

This dissertation presents the Hamiltonian description of radiation phenomena in three classical field theories based on the wave equation: scalar field theory, electrodynamics and linear gravity.

We proposed a new approach to the problem of radiation which involves the study of field evolution in the region of spacetime bounded by a light cone. We formulated Hamiltonian dynamics of initial data on a part of hyperboloid within the light cone, supplemented by an appropriate data on its boundary. The Hamiltonian function was composed of the “Trautman-Bondi energy” assigned to data on the hyperboloid, and completed by the “radiation energy” related to data on the light cone. Performing a limiting transition, where the cone is shifted to infinity, we obtained a real radiation problem expressed by Hamiltonian dynamics of the initial data on the entire hyperboloid and at the light infinity (called the *scri*). In particular, we achieved the Hamiltonian interpretation of the Trautman-Bondi energy, which was defined for each of investigated theories. This approach also allowed us to identify easily the *radiation data* at the light infinity. We obtained them from the data on the light cone considering limiting transition.

We used our method of description of radiation in the massless scalar field theory analyzing the case of two- and four-dimensional Minkowski spacetime separately. We proposed a universal approach, which solves the problem of compatibility conditions for Cauchy data on the hyperboloid and the radiation data on the *scri* (often called *corner conditions*), necessary to obtain a well defined Hamiltonian dynamical system. We found the Hamiltonian function generating dynamics and examined canonical structure on the space of radiation data on the *scri*. Additionally, in case of a two-dimensional Minkowski spacetime we derived explicit formulas for the transformation between the data on the hyperboloid and the data on the light cone. Using these formulas we proved that this transformation is a symplectomorphism.

In the Hamiltonian description of radiation in electrodynamics we used the fact that the whole evolution of the electromagnetic field can be reduced to gauge invariant scalar

quantities. We represented the field dynamics on the hyperboloid and on the light cone through these quantities. We then proved that their dynamics is equivalent to the dynamics of the two independent massless scalar fields. This equivalence helped us to define correctly the Hamiltonian evolution of the entire system, which means evolution of the initial data on a part of the hyperboloid and on the boundary of the light cone, and identify radiative degrees of freedom of the electromagnetic field on the *scri*.

In linear gravity we began an analysis of radiation from the derivation of the field equations on a hyperboloid, we received ones from linearization of the Einstein equations in the ADM form. Then we used the fact that the dynamics can be reduced to two independent degrees of freedom represented by gauge invariant quantities. In this thesis we considered one degree of freedom, so called *axial* degree of freedom. We showed that dynamics of axial degree of freedom is equivalent to dynamics of the massless scalar field. This allowed us to exploit the results obtained in the theory of scalar field and electrodynamics to formulate linear gravity as a Hamiltonian dynamical system, which describes the field dynamics on the hyperboloid and on the *scri*.

Warsaw, April 2013